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Accounting for the effects of gas species on piston gauge calculated area: absolute mode

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Abstract. A model has been developed, and tested experimentally, to account for the apparent dependence of the effective cross-sectional area of pistons in gas-piston-gauge pressure standards on the particular gas with which the gauge is operated, in the absolute mode. The model treats a pressure drop or loss that depends on the pressure difference across the gauge ($P_1 - P_2$), the fall rate of the piston, the density of the gas and the viscosity of the gas. The model was tested using helium, neon, argon, nitrogen and krypton, and several values of ($P_1 - P_2$). An algorithm was developed for calculating the piston cross-sectional area. The experimental results confirmed the model and resulted in a reduction in measurement uncertainty arising from this effect by approximately one order of magnitude.

1. Introduction

The calculated cross-sectional area of pistons in gas-piston-gauge pressure standards has been found to depend on the particular gas that flows through the annular space between piston and cylinder. The apparent difference in area between helium and nitrogen can be greater than 20 parts per million (PPM) in the absolute mode. This effect contributed the largest remaining uncertainty in the pressure generated or measured using particular gas-piston gauges. It was the objective of the present work to develop and experimentally test a model that accounts for the discrepancy and to develop an algorithm for calculating piston cross-sectional area.

A gas-piston gauge consists of a piston fitting into a matching cylinder. The annular space between the piston and cylinder is filled with a gas. The piston is loaded with known mass artefacts, i.e. weights. The force due to pressure acting on the base of the piston is balanced by gravitational force on the piston and its load of weights so that at equilibrium the piston floats. In operation, the piston and weights are rotated to relieve friction and to ensure concentricity. The piston falls slowly and the gas flows up through the annular space between the piston and cylinder. The fall rate is measured. Hereinafter the 'piston cross-sectional area' will be abbreviated to 'piston area'.

2. Model and derivation of equations

The cross-float procedure for comparing piston gauges is described in a paper by Heydemann and Welch (1975). The experimental configuration is illustrated in figure 1. In this configuration a test gauge or the gauge to be calibrated (indicated by C) is in parallel with a reference gauge (indicated by R). At the level indicated on the figure a pressure balance exists.

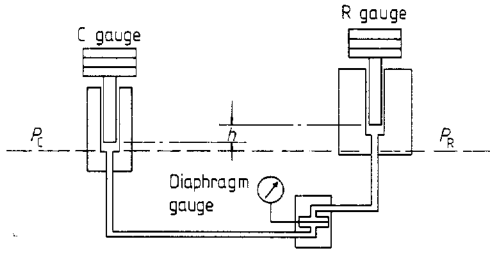

Figure 1. Sketch of cross-float configuration.

In the treatment in this paper, the gas-piston gauges are operated in the absolute mode, i.e. the piston and weights are enclosed by a bell jar. The bell jar is evacuated by a vacuum pump.

In the model adopted in the present work the area of interest is the mean cross-sectional area of the piston. Corrections are made for thermal expansion and contraction. Elastic distortion is negligible in the range of pressure of present interest. The conventional concept of piston gauge effective area is *not* used here. A pressure drop, ΔP, is introduced for each gauge to account for the *apparent* change in area with gas species. The ΔP values are also considered to include errors in measurements of various quantities.

The pressure at the base of the C gauge, P_c, is given by

$$P_c = (M_c g/A_c) + P_{bc} - P_H - \Delta P_c \qquad (1)$$

where M_c is the mass load on the piston, g is the acceleration due to gravity, A_c is the mean area of the piston at temperature t, P_{bc} is the back pressure above the gauge in the absolute mode, P_H is the pressure equivalent of an upward force on a hat-shaped structure above the weight stack and ΔP_c is the pressure drop introduced above.

At the same level, the pressure on the R-gauge side, P_r, is given by

$$P_r = (M_r g/A_r) + P_{br} + P_h - \Delta P_r \qquad (2)$$

where P_h is the pressure difference due to a difference in the heights of the two gauges above an arbitrary horizontal plane; all other quantities correspond to similar quantities in equation (1). P_H is not included in equation (2) since its magnitude for the reference gauge is small, estimated to be less than 0.2 part per million.

An imbalance in pressure between P_c and P_r is indicated, by the diaphragm gauge in the figure, as P_u; thus

$$P_c = P_r + P_u. \qquad (3)$$

By substituting equations (1) and (2), equation (3) becomes

$$(M_c g/A_c) + (P_{bc} - P_H) - \Delta P_c = (M_r g/A_r) + (P_{br} + P_h + P_u) - \Delta P_r. \qquad (4)$$

Designating $(\Delta P_c - \Delta P_r)$ as DP,

$$\mathrm{DP} = (\Delta P_c - \Delta P_r) = [(M_c g/A_c) - (M_r g/A_r)] + (P_{bc} - P_{br} - P_H - P_h - P_u). \qquad (5)$$

Since the pressure balance exists at the experimental temperature t, the piston areas A_c and A_r are expressed as

$$A_c = A_c(t) = A_c(23)[1 + 2\alpha_c(t-23)] = A_c(23)(\mathrm{TC})_c \qquad (6)$$

and

$$A_r = A_r(t) = A_r(23)[1 + 2\alpha_r(t-23)] = A_r(23)(\mathrm{TC})_r \qquad (7)$$

where α_c and α_r are coefficients of thermal expansion. The value of α_c for 'Vasco supreme' of which the pistons and cylinders of C gauges are constructed is 9.405×10^{-6} °C^{-1}; the value of α_r for tungsten carbide of which the pistons and cylinders of the R gauges are constructed is $4.144 \times 10^{-6}\ {}^\circ\mathrm{C}^{-1}$.

By substituting equations (6) and (7) and rearranging equation (5), $A_c(23)$ is expressed as

$$A_c(23) = (M_c g)[1 + 2\alpha_c(23 - t)]/[\mathrm{DP} + ((M_r g)/A_r^0(23)(\mathrm{TC})_r) - (P_{bc} - P_{br} - P_H - P_h - P_u)]. \quad (8)$$

Equation (5) is used to calculate DP from measurements for the various parameters and reference values of $A_c(23)$ and $A_r(23)$. Equation (8) is used to calculate $A_c(23)$ from measurements of the various parameters, an *estimate* of DP, and the reference value $A_r^0(23)$.

By setting $\mathrm{DP} = 0$ in equation (8), one arrives at an equation of the form that is conventionally used to calculate $A_c(23)$. Using the conventional equation, calculated values of $A_c(23)$ have been found to depend on the particular gas passing through the space between the piston and cylinder. The gas-species dependence is illustrated in table 1. The difference between the value of area for nitrogen and that for helium is 23.4 parts per million of area.

In the development that follows, a source of the major portion of the pressure drops (ΔP) will be identified, the relationship between ΔP and relevant gas properties will be developed and used to calculate values of DP, the DP values will be used to calculate values of $A_c(23)$.

Table 1. Typical dependence of conventionally calculated area on gas species, absolute mode.

Gas	Area (10^{-5} m^2)	Departure from helium value (PPM)
Helium	8.107752	—
Neon	8.107805	6.5
Argon	8.107929	21.8
Nitrogen	8.107942	23.4
Krypton	8.107977	27.8
Carbon dioxide	8.108018	32.8

3. Hagen–Poiseuille and pressure drop or loss

The space between the piston and cylinder of a piston gauge approximates an annulus of outer radius b', inner radius a' and length l'. In the operation of the gauge, the flow of gas through this space approximates to the flow through a channel of concentric cylindrical annular section.

In 1839, Hagen (1839) investigated the laminar flow of water through brass tubes of various diameters and expressed the pressure head, h, of his supply tank as a function of the mass of water flowing out per second, W (Prandtl and Tietgens 1934)

$$h = h_1 + h_2 = aW + bW^2 \quad (9)$$

where a and b were constants for each tube. Hagen observed that the part $h_2 = bW^2$ of the total head was used to impart kinetic energy to the fluid. Prandtl and Tietgens (1934) introduced the mean velocity $\bar{u}$ rather than W, the pressure difference $\Delta p = \rho g h$, and the viscosity of the fluid μ to transform equation (9) to

$$\Delta p = \Delta p_1 + \Delta p_2 = (8\mu l \bar{u}/r^2) + (2.7\rho\bar{u}^2/2) \quad (10)$$

in CGS units, where l is the length of the tube, r is the radius of the tube and ρ is the density of the fluid. The second terms on the right-hand side of equation (10) represent the pressure difference required to impart kinetic energy to the fluid.

Poiseuille (1840) independently found the same law for the laminar flow of water through capillary tubes of glass, although his formulation lacked the pressure difference required to impart kinetic energy to the fluid. An equation of the form of (10) is generally referred to as the Hagen–Poiseuille equation.

For a concentric cylindrical annulus with a pressure difference of $(P_1 - P_2)$ across the annulus, equation (10) can be written

$$(P_1 - P_2) = (\mu\bar{u}/B) + K\rho\bar{u}^2 \quad (11)$$

where B is a factor that incorporates the geometry of the annulus and K is a constant. If the second term on the right-hand side of equation (11) were much smaller than the first,

$$\bar{u} \simeq B(P_1 - P_2)/\mu \quad (12)$$

and the second term becomes

$$K\rho\bar{u}^2 = [KB(P_1 - P_2)/\mu](\rho\bar{u}) = K'(P_1 - P_2)(\bar{u}\rho/\mu). \quad (13)$$

The rationale for choosing to derive this form of the second term will be given later.

The pressure difference represented by equation (13) is conventionally associated with the kinetic energy required to create a parabolic velocity distribution in the annulus (in this case) near the entrance (Santeler 1986). The net effect of the transition to the constant velocity profile is to lower the flow rate for a given pressure difference, $(P_1 - P_2)$, across the channel or, alternatively, to increase the pressure drop required for a given flow rate when compared with the Poiseuille case (Worden 1962).

The equations in this section apply to laminar viscous flow of fluids including gases. The question arises whether for the absolute mode (that is, with the space above the piston evacuated) of operation of piston gauges, the flow through the annular channel can be treated as viscous. Conventionally, the gas flow is considered to change from viscous at the inlet of a channel to a transition state in the middle and finally to molecular at the vacuum exit (Santeler 1986). Santeler (1986), discussing a leak in a vacuum system in which the pressure on the vacuum side of the leak was 0.013 Pa and the external pressure was 101 325 Pa, concluded that the gas flow in the leak was viscous. The piston gauge system would seem to be sufficiently similar to adopt a laminar viscous flow model on which to interpret experimental results, and to use the results to ascertain the applicability of the model. Other considerations have led to the same conclusion.

4. Piston-gauge model flow

For the piston gauge treated here, the model flow is laminar viscous flow of gas through the annulus between piston and cylinder. A loss or pressure drop *of the form*

$$K'(P_1 - P_2)\bar{u}\rho/\mu$$

is taken to account for the major part of the pressure drop, ΔP, introduced above to account for the dependence of conventionally calculated piston area on the gas species flowing through the annulus. In the present treatment the areas are mean areas of the pistons, accounting for thermal expansion and contraction. The effects of elastic distortion are negligible for the range of pressure investigated in this work. The conventional piston gauge concept of effective area is not used here.

5. Experimental details

In equation (13), K' is a coefficient with dimensions of length. K' varies from piston gauge to piston gauge. The mean velocity $\bar{u}$ is proportional to the fall rate (FR) of the piston; equation (13) becomes

$$K\rho\bar{u}^2 = K''(P_1 - P_2)(\mathrm{FR}\rho/\mu). \quad (14)$$

Experimental determinations of K'' have been made for three piston gauges in the present work.

Two sets of experiments were performed to investigate the validity of the approach outlined above. In the first set of experiments, each of two piston gauges of the type conventionally calibrated at the National Bureau of Standards, designated C_1 and C_2, was cross-floated in the absolute mode with a reference gauge, R. Helium, neon, argon, nitrogen, krypton and carbon dioxide were used as fluids. Helium and nitrogen are in general use in piston gauges. The other gases were chosen primarily on the basis of the value of the ratio ρ/μ which appears in equation (14). On a scale of ($\mathrm{FR}\rho/\mu$), neon and argon fall between helium and nitrogen; krypton and carbon dioxide extend the range beyond nitrogen (Jones 1978, 1984, Hellmans *et al* 1974). Gauges C_1 and C_2 were separately cross-floated with gauge R with a pressure difference of $(P_1 - P_2)$ across the gauges for each of the six gases. Gauge C_2 was cross-floated also with gauge R at other values of $(P_1 - P_2)$, 79 190 Pa and 127 100 Pa, using helium, nitrogen and krypton. Measurements of t, PB, PU, calculated values of P_h and estimates of values of P_H were used in calculations.

In a second set of experiments, each of the piston gauges was operated in the absolute mode against the NBS gas thermometer manometer, at each of three pressures, 96 000 Pa, 62 000 Pa and 27 000 Pa at temperatures near the reference temperature, 23 °C. At the 62 000 Pa pressure the gas used was helium, at the other two pressures the gases used were helium and nitrogen. The conventionally calculated areas at 23 °C for gauges R, C_1 and C_2 are listed in table 2. The number of measurements N, the number of sets of measurements and an estimate of the standard deviation, SD (in PPM), are included. The calculated areas for helium at 27 000 Pa are used as reference values in this work: $A_R^0(23) = 3.358\,210\,63 \times 10^{-4}\ \mathrm{m}^2$, $A_{C_1}^0(23) = 8.107\,710\,96 \times 10^{-5}\ \mathrm{m}^2$ and $A_{C_2}^0(23) = 8.107\,558\,16 \times 10^{-5}\ \mathrm{m}^2$.

Table 2. Mean areas of three piston gauges in the absolute mode determined from gas thermometer manometer measurements.

Gauge	$(P_1 - P_2)$ (kPa)	Gas	A (10^{-5} m^2)	SD (PPM)	N	Sets
R	27	He	33.582 106 3	0.65	40	4
	27	N	33.582 233 8	0.68	52	6
	62	He	33.582 131 8	0.76	60	4
	96	He	33.582 098 5	1.2	93	6
	96	N	33.582 276 7	0.55	56	4
C_1	27	He	8.107 710 96	2.4	61	5
	27	N	8.107 874 66	1.7	40	5
	62	He	8.107 756 09	0.79	59	4
	96	He	8.107 796 13	0.46	32	2
	96	N	8.108 034 23	0.67	60	4
C_2	27	He	8.107 558 16	1.5	62	4
	27	N	8.107 713 14	1.0	28	4
	62	He	8.107 606 57	0.55	60	4
	96	He	8.107 658 05	0.70	79	5
	96	N	8.107 872 21	0.68	60	4

6. Treatment of data

The data generated by the first set of experiments and the reference values of areas from the second set of experiments are used in equation (5) to calculate values of DP. In the cross-float configuration, the model adopted above treats a *combination* of kinetic-energy-related pressure drops or losses in two gauges as the major part of DP:

$$\mathrm{DP} = (\Delta P_C - \Delta P_R) \simeq K_C''(P_1 - P_2)\mathrm{FR}_C(\rho_C/\mu_C) - K_R''(P_1 - P_2)\mathrm{FR}_R(\rho_R/\mu_R). \quad (15)$$

Since the temperature of the gas is nearly equal in the two gauges, the densities (Worden 1962) and viscosities (Jones 1978, 1984) can be set equal, thus

$$\mathrm{DP} \simeq [(P_1 - P_2)\rho/\mu](K_C''\mathrm{FR}_C - K_R''\mathrm{FR}_R). \quad (16)$$

The loss in the reference gauge is inferred, from the relative magnitude of the effect of gas species on conventionally calculated area, to be smaller than the losses for the other two gauges. Therefore, as a preliminary indication (only) of the qualitative validity of the model, DP is plotted against $(P_1 - P_2)\mathrm{FR}_C(\rho/\mu)$, in figure 2. The plot is for gauge C_1 in the absolute mode at 102 600 Pa. The bars on the points represent ± 1 PPM of area. The curve drawn through the points is seen to be smooth, representing a monotonic relationship and qualitatively confirming the validity of the quantity plotted on the abscissa. Note that a straight line could be fitted to the first four points (helium, neon, argon and nitrogen). The points would fit the line very precisely, within several tenths of a part per million.

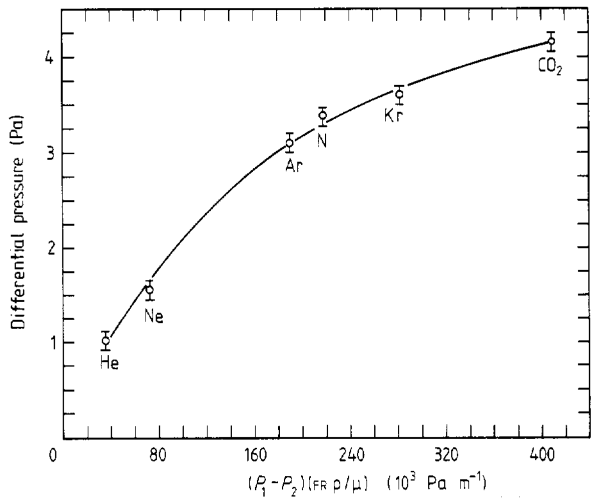

Figure 2. Plot of DP against $(P_1 - P_2)(\mathrm{FR}\rho/\mu)$ for gauge C_1 in the absolute mode for $(P_1 - P_2) = 102\,600$ Pa. The bars on the points represent ± 1 PPM.

Values of K_R'', K_{C_1}'' and K_{C_2}'' have been calculated using the data generated in the second set of experiments. By separating and equating terms on both sides of equation (15), one can approximate ΔP_R

$$\Delta P_R \simeq K_R''(P_1 - P_2)\mathrm{FR}_R(\rho/\mu). \quad (17)$$

By rearranging equation (17), one can approximate K_R''

$$K_R'' \simeq [\Delta P_R/(P_1 - P_2)]/[\mathrm{FR}_R(\rho/\mu)]. \quad (18)$$

The ratio of the calculated area A_R in table 1 to the reference area, $A_R^0(23)$, less unity is an approximation to $\Delta P_R/(P_1 - P_2)$. The slope of a linear fit of the approximation against $[\mathrm{FR}_R(\rho/\mu)]$ is an estimate of K_R''. The value thus determined using the data generated by the second set of experiments is 12.2×10^{-6} m.

The right-hand side of equation (17) is now referred to as ΔP_R, the kinetic-energy-related drop in gauge R. Having determined K_R'' experimentally, values of ΔP_R are added to corresponding values of DP to arrive at ΔP_C, which represents the kinetic-energy-related pressure drop in C_1 or C_2, alone. Thus, $\text{DP} = \Delta P_C - \Delta P_R$, $\Delta P_C = \text{DP} + \Delta P_R$.

Although, as indicated in figure 2, the results for carbon dioxide fit the model, the operation of the piston gauges was sufficiently sluggish for carbon dioxide that the imprecision was considered to be atypical of that for the other five gases. Therefore, detailed analysis was confined to data for helium, neon, argon, nitrogen and krypton.

The variation in measured fall rate, FR, for each set of measurements was several per cent of FR. Thus, a representation of ΔP in the form $K\rho\text{FR}^2$ would include measurement imprecision nearly double that in the $K''(P_1 - P_2)(\text{FR}\rho/\mu)$ representation. The group $(\text{FR}\rho/\mu)$ is proportional to a Reynolds number: $R = D\rho\bar{u}/\mu$, where D is a characteristic dimension.

7. Results

Using the experimental measurements of various quantities on the right-hand side of equation (5) and reference values of A_R and A_C, the equation provides values of DP. In order to use equation (8) to calculate A_C at the reference temperature, 23 °C, a measure of DP is required. This measure of DP is obtained from an equation polynomial in $(P_1 - P_2)(\text{FR}_C\rho/\mu)$ fitted by least squares. Alternatively, ΔP_R is calculated using the experimental value of K''_R in equation (17) and added to DP to arrive at ΔP_C; ΔP_C is then fitted against $(P_1 - P_2)(\text{FR}_C\rho/\mu)$. From the measure of ΔP_C, calculated using the fitted equation, ΔP_R is subtracted, providing a measure of DP to be inserted in equation (8) to calculate $A_C(23)$. For the data in the present work, either of these methods can be used and the resulting values of $A_C(23)$ are of essentially equal precision. The first is, of course, simpler to use. The polynomial equations are usually quadratic in $(P_1 - P_2)(\text{FR}\rho/\mu)$.

For gauge C_1 in the absolute mode, at $(P_1 - P_2) = 102\,600$ Pa, the values of $\Delta P_C = \text{DP} + \Delta P_R$ were fitted to an equation quadratic in $(P_1 - P_2)(\text{FR}_C\rho/\mu)$. Values of calculated ΔP_C less ΔP_R were inserted for DP in equation (8) to calculate values of $A_C(23)$, for helium, neon, argon, nitrogen and krypton. The results are listed in table 3. The mean value of $A_C(23)$ was $8.107\,710\,8 \times 10^{-5}$ m^2 with an estimate of standard deviation, SD, of 3.5×10^{-11} m^2 corresponding to a relative SD of 0.43 part per million of area. The reference value, $A_C^0(23)$, for gauge C_1 was $8.107\,710\,96 \times 10^{-5}$ m^2.

Table 3. Area for gauge C_1 in absolute mode at $(P_1 - P_2) = 102\,600$ Pa.

Gas	$A_C(23)(10^{-5}\ \text{m}^2)$
Helium	8.1077140
Neon	8.1077055
Argon	8.1077105
Nitrogen	8.1077140
Krypton	8.1077100
	$\bar{A}_C(23) = 8.1077108$
	SD $= 0.0000035$
	Rel. SD $= 0.43$ PPM
	$A_C^0(23) = 8.10771096$

For gauge C_2 in the absolute mode at three different values of $(P_1 - P_2)$, 79 190 Pa, 102 600 Pa and 127 100 Pa, values of ΔP_C were fitted to an equation cubic in $(P_1 - P_2)(\text{FR}_C\rho/\mu)$. Values of $A_C(23)$ were calculated for 9 of the 11 points. The point for argon at 102 600 Pa was excluded as the DP was obviously too large. The fall rate for the low-pressure helium point was inconsistent and it was consequently excluded. The results for the 9 points are listed in table 4. The mean value of $A_C(23)$ was $8.107\,558 \times 10^{-5}$ m^2 with a SD of 1.0×10^{-10} m^2 corresponding to a relative SD of 1.2 parts per million of area. The reference value, $A_C^0(23)$, was $8.107\,558\,2 \times 10^{-5}$ m^2.

As these values indicate, the model and algorithm developed in this work result in very precise values of $A_C(23)$.

In figure 3, values of DP are plotted against $(P_1 - P_2)(\text{FR}_C\mu/\rho)$ for gauge C_1 in the absolute mode at $(P_1 - P_2) = 102\,600$ Pa. On the same figure ΔP_R calculated using K''_R and ΔP_C, the sum of DP and ΔP_R, are also plotted. ΔP_R is the kinetic-energy-related pressure drop in the reference gauge R, ΔP_C is the corresponding pressure drop in gauge C_1, and DP is the net effect of these two quantities in the cross-float configuration, $\text{DP} = \Delta P_C - \Delta P_R$. This figure illustrates the composition of the pressure drop which accounts for the former apparent dependence of $A_C(23)$ on gas species, and the decomposition into separate pressure drops in the two gauges in the cross-float configuration.

Again, a straight line could be fitted very precisely to the first four points of the DP and ΔP_C plots.

Table 4. Area for gauge C_2 in absolute mode at three values of $(P_1 - P_2)$.

$(P_1 - P_2)$(kPa)	Gas	$A_C(23)(10^{-5}\ \text{m}^2)$
102.6	Helium	8.1075620
	Neon	8.1075625
	Nitrogen	8.1075715
	Krypton	8.1075418
79.1	Nitrogen	8.1075655
	Krypton	8.1075500
127.1	Helium	8.1075470
	Nitrogen	8.1075670
	Krypton	8.1075580
		$\bar{A}_C(23) = 8.107558$
		SD $= 0.000010$
		Rel. SD $= 1.2$ PPM
		$A_C^0(23) = 8.1075582$

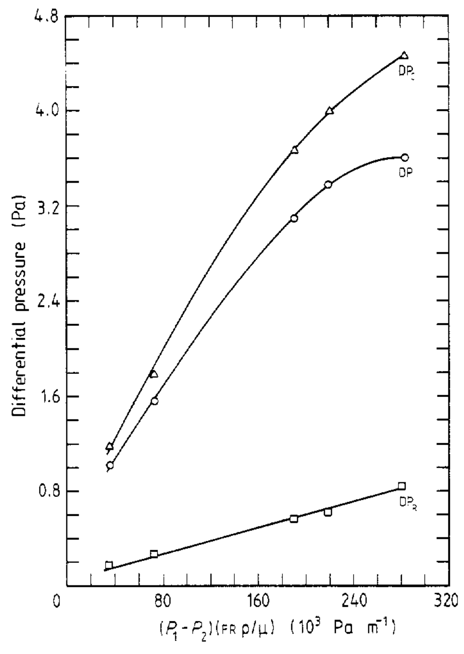

Figure 3. Plots of DP, DP_C and DP_R against $(P_1 - P_2)(\text{FR}\rho/\mu)$ for gauge C_1 in the absolute mode at $(P_1 - P_2) = 102\,600$ Pa.

8. Summary and conclusions

In the present work, the apparent dependence of the effective area of pistons in gas-piston gauges, operated in the absolute mode, on gas species, was investigated. A model was adopted in which the area of interest is the mean area of the piston. Pressure differences, ΔP values, are introduced to account for the major part of the gas-species dependence. The major part of the ΔP is attributed to the pressure drop or loss due to the kinetic energy required to establish the velocity profile in the annular space between the piston and cylinder. The pressure drop is shown to depend on the pressure difference across the gauge, $(P_1 - P_2)$, the fall rate of the piston, FR, the density of the gas, ρ, and the viscosity of the gas, μ. For the individual gauge, a pressure drop is shown to be a function of $(P_1 - P_2)(\mathrm{FR}\rho/\mu)$. For two gauges in a cross-float configuration the net kinetic-energy-related pressure drop, DP, is equal to the difference between the pressure drop for the gauge being calibrated and the pressure drop for the reference gauge.

Values of DP (or ΔP_C) are determined experimentally, for helium, neon, argon, nitrogen and krypton, and fitted against $(P_1 - P_2)(\mathrm{FR}\rho/\mu)$. Calculated values of DP from the fitted equation are used to calculate $A_C(23)$, the area of the piston in gauge C_1, at the reference temperature, 23 °C.

The calculated values of $A_C(23)$ are very precise, with a relative estimate of standard deviation of 1.2 parts per million of area or less.

It should be emphasised that the piston areas calculated in this work are the real physical areas of the pistons, not the effective areas. Also, the magnitude of the effect of gas species found for the C gauges in this work could be smaller, as illustrated for the reference gauge in figure 3, or larger for other piston gauges. The effect was sufficiently large for the C gauges that the detailed analysis and decomposition of the effect could be made very effectively.

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